3

Theory and Measurement of the Elasticity of Substitution in International Trade

The theoretical presumption of relatively high price elasticities in international trade has frequently not been borne out in empirical studies using ordinary least squares analysis of time series. There has as a consequence been a search for conceptual alternatives, the measurement of which would yield results more in concert with the a priori presumption. One such alternative commonly employed has been the "elasticity of substitution," which is defined simply as the percentage change in relative quantities demanded divided by the percentage change in relative prices

$$e = \frac{\partial(q_1/q_2)}{q_1/q_2} \div \frac{\partial(p_1/p_2)}{p_1/p_2} = \frac{\partial(q_1/q_2)}{\partial(p_1/p_2)} \cdot \frac{p_1 q_2}{p_2 q_1} = \frac{\partial \log (q_1/q_2)}{\partial \log (p_1/p_2)}$$

where q_1 and q_2 are exports from two competing supply sources to some third market (perhaps the rest of the world), and p_1 and p_2 are their respective prices. The partial derivatives in the definition are the mathematical analogue of the economist's *ceteris paribus* assumption. In the case of the utility analysis of demand, it is real income that is being held constant. We will see that when applied statistically the *ceteris paribus* phrase may be conveniently implemented in the present context with regard to money income and other prices.

It is evident that if q_1 and q_2 are absolute complements (such as right shoes and left shoes), no change can occur in q_1/q_2 and e will be zero. Whereas if q_1 and q_2 are perfect substitutes, consumers will purchase only the lower-priced item, in which case e will be $-\infty$ at $p_1 = p_2$ and zero elsewhere. Our interest is in cases lying between these limits, and we naturally would like to be able to specify ranges of values that could be considered large, medium, or small. Unfortunately, as we will see, values of the elasticity of substitution other than zero and $-\infty$ pose serious problems in interpretation. Worse yet, the method used to estimate the elasticity of substitution may yield a result unrelated to the valid theoretical concept. In general, we look on the estimation of the elasticity of substitution with considerable skepticism. We will

establish rather strong doubts concerning both the statistical method and the theoretical usefulness of the concept.

We will address ourselves accordingly in the following discussion primarily to three fundamental questions: (1) What is the theoretical foundation of the elasticity of substitution? (2) Can we devise a method of analyzing the data which is likely to disclose the numerical value of the theoretical concept? (3) Assuming we knew the true value of the elasticity, how would it be interpreted and used? In most of what follows, we shall have time series in mind. While cross-section estimation will be treated explicitly later in the chapter, it should be emphasized that many of the points to be made concerning time series apply equally to cross sections.

THEORETICAL FOUNDATION

The elasticity of substitution is rigorously defined with respect to movement along a single indifference curve.[1] Such a situation is depicted in Figure 3.1, with II as the indifference curve, AB as the original price line, and $A'B'$ as the final price line.

In this case the elasticity may be estimated by

$$e = \frac{\Delta(q_1/q_2)}{q_1/q_2} \div \frac{\Delta(p_1/p_2)}{p_1/p_2} = \frac{q_1/q_2 - q_1'/q_2'}{q_1/q_2} \div \frac{A'/B' - A/B}{A/B}$$

In general, the value of e will depend on the particular indifference curve that is selected, as well as on the value of p_1/p_2. Such a situation is cumbersome from a theoretical point of view and, as will be shown, can be crippling to any empirical work. It will behoove us, therefore, to determine the conditions under which the value of the elasticity of substitution depends on the value of p_1/p_2 alone. The proper requirement suggests itself immediately: the slope of the indifference curve must depend on q_1/q_2 and not on any scale factor.[2] Another way of saying this is that the income elasticities of the two goods are identical.

For the sake of realism, we should take other goods besides q_1 and q_2 into account. The additional requirements affecting e in such an event are

[1] See Morrissett [17] for a historical sketch and careful theoretical discussion of the concept. Another source of insight is Morgan and Corlett [16].

[2] See Morrissett [17] for a proof. Linear homogeneous functions have this property. Monotone transformations of linear homogeneous functions do as well.

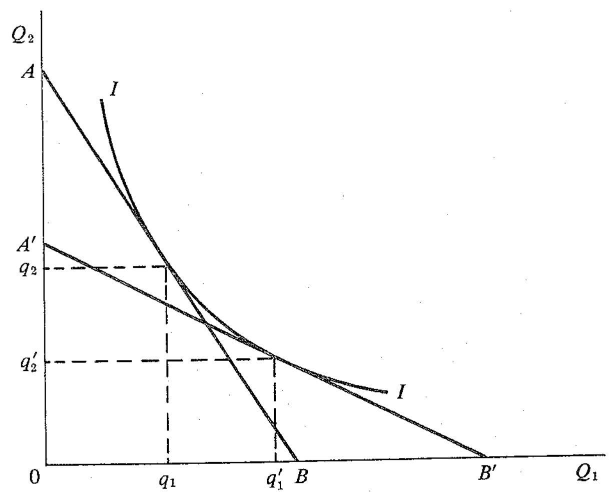

FIGURE 3.1

The Elasticity of Substitution
Along a Single Indifference Curve

similar to the one just mentioned: the proportional responses of q_1 and q_2 to changes in the levels of all other variables should be equal.[3]

The point just made is troublesome since it seems as if we are trying to say that q_1 and q_2 are the same commodity. Yet if they were, the indifference curves would be straight lines and the analysis would degenerate. The most we can say therefore is that e will depend only on relative prices when the two commodities in question are so similar that the reaction of demand for each to all other economic variables is identical, yet at the same time are dissimilar enough to induce the purchase of some of both.

The measurement of the elasticity of substitution has invariably been attempted using a log-linear regression

$$\log \frac{q_1}{q_2} = a + b \log \frac{p_1}{p_2} \tag{3.1}$$

where the coefficient b is the elasticity of substitution, which by virtue of the logarithmic form is constrained to be constant. We can consider this con-

[3] To put this in another way, one can think of the indifference curves in Figure 3.1 drawn with the other variables held constant. The condition for e to depend on p_1/p_2 alone is then that the indifference map appears the same for all choices of the other variables.

stancy to result not from explicit assumption but rather from ignoring the problems surrounding the choice of functional form on the basis of the general relationship

$$\frac{q_1}{q_2} = f\left(\frac{p_1}{p_2}\right) \tag{3.2}$$

Besides the fact that Equation (3.1) is open to criticism for ignoring the problem of functional form, an important additional criticism concerns the implicit assumption that q_1/q_2 is dependent only on p_1/p_2, which requires the rather strong symmetry assumptions discussed above.

Thus suppose for the moment that the symmetry assumptions do not hold, yet we persist in running the regression suggested in Equation (3.1). As evident from Figure 3.2, the resulting estimate need bear no relationship

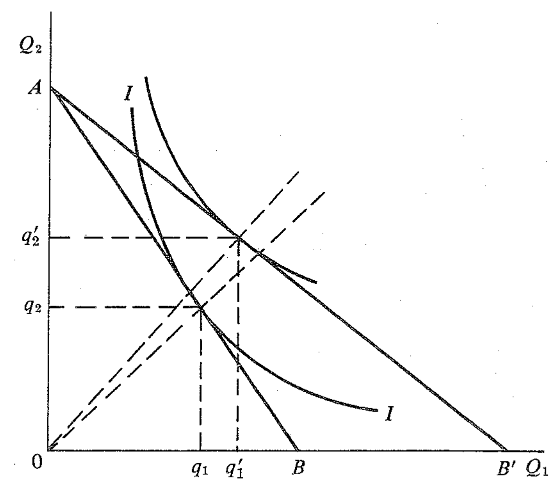

FIGURE 3.2

The Elasticity of Substitution
Between Two Indifference Curves

to the theoretical concept. That is, a fall in the price of q_1, indicated by the shift in the price line AB to AB', has resulted in a fall in q_1/q_2 to q_1'/q_2'. The measured elasticity of substitution would thus turn out to be positive rather than negative in this instance and would provide no insight into the true elasticity of substitution defined along the indifference curve.[4]

[4] See Stern and Zupnick [22, pp. 484–86] for a similar demonstration using partial equilibrium demand and supply schedules.

To this point our discussion has implicitly assumed that the importing country has a well-behaved indifference map. In view, however, of the well-known conceptual difficulties involved in such community indifference maps, it may be more fruitful to examine the elasticity of substitution in the framework of conventional demand analysis. Let us write the following export-demand functions

$$q_1 = f(p_1, p_2, y, p_n) \quad \text{and} \quad q_2 = g(p_1, p_2, y, p_n) \tag{3.3}$$

where y is money income in the importing country and p_n is the general price level in this country of commodities other than 1 and 2, including perhaps competing imports. For purposes of simplification we will assume constant-elasticity approximations to (3.3)

$$q_1 = a\, p_1^{\alpha_1} p_2^{\alpha_2} y^{\alpha_y} p_n^{\alpha_n} \quad \text{and} \quad q_2 = b\, p_1^{\beta_1} p_2^{\beta_2} y^{\beta_y} p_n^{\beta_n} \tag{3.4}$$

where the α's and β's refer to the elasticities of the respective variables. We can then write

$$\frac{q_1}{q_2} = \frac{a}{b} \frac{p_1^{\alpha_1-\beta_1}}{p_2^{\beta_2-\alpha_2}} y^{\alpha_y-\beta_y} p_n^{\alpha_n-\beta_n} \tag{3.5}$$

The elasticity of substitution may now be conveniently defined holding money income y and other prices p_n constant.

It should be evident from Equation (3.5) that q_1/q_2 will be functionally related to p_1/p_2 only if the exponents of the price variables are equal

$$e = \alpha_1 - \beta_1 = \beta_2 - \alpha_2$$

or

$$\alpha_1 + \alpha_2 = \beta_1 + \beta_2 \tag{3.6}$$

Equation (3.6) asserts that the sum of the direct and cross elasticities of demand be the same for each commodity.[5] This is quite similar to the symmetry conditions discussed earlier in connection with the utility analysis, and the same conclusion holds. Commodities q_1 and q_2 must be quite similar but not too similar.

It will be further evident from Equation (3.5) that there are two variables, y and p_n, that do not appear in the regression equation (3.1). This is justifiable only when $\alpha_y = \beta_y$ and $\alpha_n = \beta_n$, that is, when the income elasticities of each commodity are comparable and when the cross-price elasticities with respect to other goods are also comparable.

The points just made are essentially empirical problems, and it would seem advisable to test their validity in a regression of the form

$$\log\left(\frac{q_1}{q_2}\right) = a + b_1 \log p_1 + b_2 \log p_2 + c \log y + d \log p_n \tag{3.7}$$

The hypothesis represented by Equation (3.6) would then be examined by testing whether $b_1 = -b_2$. Similarly, $\alpha_y = \beta_y$ and $\alpha_n = \beta_n$ could be examined by testing whether $c = 0$ and $d = 0$.[6]

The question arises again as to what happens if we insist on regressions of the form (3.1) when the condition (3.6) is unwarranted. The answer is that the concept of the elasticity of substitution as a demand phenomenon degenerates since the observed value depends on the particular paths taken by the individual prices. Suppose, for example, that $\alpha_1 - \beta_1 = -1.0$, $\beta_2 - \alpha_2 = -0.1$, $\Delta p_1/p_1 = 0.9$, and $\Delta p_2/p_2 = 1.0$. Since p_1 has risen by a smaller percentage than p_2, the ratio p_1/p_2 has fallen, and we expect q_1/q_2 to rise. However, from Equation (3.5) we see that

$$\begin{aligned}\frac{\Delta(q_1/q_2)}{(q_1/q_2)} &= (\alpha_1 - \beta_1)\frac{\Delta p_1}{p_1} - (\beta_2 - \alpha_2)\frac{\Delta p_2}{p_2} \\ &= 1.0(0.9) - (-0.1)(1.0) \\ &= -0.9 + 0.1 = -0.8\end{aligned}$$

Contrary to what is expected, a fall in p_1/p_2 has resulted in a fall in q_1/q_2 and the observed elasticity of substitution is positive. The observed elasticity of

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[5] Since Equation (3.4) is meant to be an approximation to the general demand equation (3.3), we should be thinking of the exponents of (3.4) as being dependent on the arguments of the functions (3.3). For example, in approximating f we will pick a particular set of values for p_1, p_2, y, and p_n. In a neighborhood around this point, f will be well approximated by $a\, p_1^{\alpha_1} p_2^{\alpha_2} y^{\alpha_y} p_n^{\alpha_n}$. Were we to select a different point, say (p_1', p_2', y', p_n'), we would no doubt have to choose different values of a, α_1, α_2, α_y, α_n to yield a good approximation. In this sense, a, α_1, α_2, α_y, and α_n depend on the values of p_1, p_2, y, p_n. In this situation an additional requirement is needed for q_1/q_2 to be functionally related to p_1/p_2. Not only must (3.6) hold, but also the values of the exponents α_1, α_2, β_1, β_2 must be either independent of p_1 and p_2 or dependent on the ratio p_1/p_2 only.

It is illuminating in this context to consider linear functions for (3.3)

$$q_1 = \alpha_1 + \beta_1 \frac{p_1}{p_2} + \gamma_1 \frac{y}{p_1} + \delta_1 \frac{p_n}{p_1}$$

$$q_2 = \alpha_2 + \beta_2 \frac{p_1}{p_2} + \gamma_2 \frac{y}{p_2} + \delta_2 \frac{p_n}{p_2}$$

These functions have the desirable property that if all prices and money income are multiplied by the same factor no change in demand occurs. Equation (3.5) then becomes

$$\frac{q_1}{q_2} = \frac{\alpha_1 + \beta_1(p_1/p_2) + \gamma_1(y/p_1) + \delta_1(p_n/p_1)}{\alpha_2 + \beta_2(p_1/p_2) + \gamma_2(y/p_2) + \delta_2(p_n/p_2)}$$

With use of this function, the elasticity of substitution depends not only on the levels of all the variables but also on the paths taken by p_1 and p_2, which will depend on supply conditions. In other words, q_1/q_2 is not functionally dependent on p_1/p_2, and the concept of the elasticity of substitution degenerates.

[6] If the variables y and p_n are excluded, the resulting estimate of the elasticity of substitution will necessarily be inefficient, statistically speaking. It will be unbiased only if y and p_n are uncorrelated with p_1/p_2. See Morgan and Corlett [16] for some not-too-encouraging experiments with an income term.

substitution will depend in general on the particular choice of $\Delta p_1/p_1$ and $\Delta p_2/p_2$, that is, on the paths taken by the individual prices.[7] An elasticity of substitution estimated during one period will hold for another period only if the paths of these variables are retraced.

The particular paths followed by the explanatory variables will depend, in general, on the interaction of all the other economic variables. But it may be noted that if it were possible to specify other economic relationships that completely determine the paths of the explanatory variables, the effect will be to make the elasticity of substitution unique. Supply functions come to mind immediately.[8] For example, it may be that monetary inflations in Countries 1 and 2 behave similarly and in consequence p_1/p_2 follows the same path. Thus, Country 1 may undergo approximately a 6 percent rate of inflation while Country 2 suffers only a 3 percent rate consistently over time. In this case, p_1/p_2 would grow consistently at a 3 percent rate, and the observed elasticity of substitution would be roughly constant over the period. Such an estimate would only be useful of course if we could be confident of a continuance of the inflation rates in the two countries. But even though it is quite possible that the interaction of demand and supply will create a close relationship between q_1/q_2 and p_1/p_2, such a relationship is only a description of the time series and not an analysis useful in understanding the underlying economic forces or in predicting future events. What can be concluded from our discussion of the theoretical foundation of the elasticity of substitution is that a regression of the form

$$\log \frac{q_1}{q_2} = a + b \log \frac{p_1}{p_2} \tag{3.8}$$

requires the following assumptions:

(i) The algebraic sum of cross and direct elasticities of demand for the two commodities must be equal.

(ii) The income and any other price elasticities of demand for the two commodities must be equal.[9] This implies roughly that the two commodities be alike in all economic respects except that they are not perfect substitutes. If they are perfect substitutes then b becomes $-\infty$ and $p_1/p_2 \equiv 1$ as long as some of both commodities is being sold. In this case, $\log p_1/p_2$ is a constant (as a is) and Equation (3.8) cannot be estimated.

[7] See the Appendix to this chapter for further discussion of this matter.

[8] Polak [20] introduces supply functions for q_1 and q_2 to calculate the observed elasticity of substitution in terms of demand and supply elasticities. Stern and Zupnick [22] discuss the elasticity of substitution within the framework of supply functions as well as demand.

[9] When Condition (ii) holds, Condition (i) can be replaced by the absence of money illusion in the demand functions

$$\begin{aligned} \alpha_1 + \alpha_2 + \alpha_y + \alpha_n &= 0 \\ \beta_1 + \beta_2 + \beta_y + \beta_n &= 0 \end{aligned} \tag{a}$$

This makes the demand functions homogeneous of degree zero so that doubling all prices and money income will not change the quantities demanded. If we also assume as in (ii) in the text that

$$\alpha_y - \beta_y = \beta_n - \alpha_n = 0 \tag{b}$$

we can subtract the two equations in (a) to get

$$\alpha_1 + \alpha_2 = \beta_1 + \beta_2 \tag{c}$$

which is the same as Equation (3.6). Therefore (i) and (ii) in the text can be replaced by (a), the absence of money illusion, and (b), identical elasticities with respect to the other variables.

The foregoing objections may be met by a regression of the form

$$\log \frac{q_1}{q_2} = a + b_1 \log p_1 + b_2 \log p_2 + c \log y + d \log p_n \tag{3.9}$$

which is what we suggest be used, with form (3.8) being avoided. However, form (3.9) has the disadvantage that data must be collected on income y and other prices p_n. Inasmuch as the coefficients c and d are likely to be small, we may on grounds of economy drop these two terms and fit

$$\log \frac{q_1}{q_2} = a + b_1 \log p_1 + b_2 \log p_2 \tag{3.10}$$

In effect, our preference for Equations (3.9) and (3.10) represents a rejection of the elasticity of substitution on theoretical grounds. The elasticity of substitution requirement that $b_1 = -b_2$ in these relations imposes assumptions we do not regard as suitable for a priori imposition upon the data. However, empirical tests of relationships (3.9) and (3.10) may prove that the elasticity of substitution is a useful approximation in certain contexts.

MEASUREMENT

Let us now turn to the question of measurement of the elasticity of substitution. Assuming that (i) and (ii) just mentioned hold, we must inquire whether a least squares regression of the form (3.8) yields a good estimate of the true elasticity of substitution: $e = \alpha_1 - \beta_1 = \beta_2 - \alpha_2$. In the previous chapter we observed that the existence of a supply relationship biases toward zero any least squares estimate of a price elasticity of demand. This is due to the fact that the error term in the demand relationship has a positive correlation with the price term. Unfortunately, the identical simultaneity problem exists with regard to a regression of the form (3.8).

That is, a disturbance to (3.8) such as a temporary shift in demand in favor of q_1 will be associated with an accommodating movement of p_1/p_2 as suppliers of q_1 raise prices to ration the available quantity, and suppliers of q_2 lower prices to eliminate accumulating stocks. Just as in the case of price elasticities, the estimate of the elasticity of substitution will be biased toward zero unless supply elasticities are infinite. The size of the bias will be large accordingly when the disturbances to (3.8) are large relative to the disturbances to the supply functions. There is a presumption, however, that the elasticity of substitution relation will be more stable than the corresponding demand relation. Disturbances to one of the demand functions in (3.3) are likely to have their counterparts in disturbances to the other demand function. Accordingly, when we divide these demand functions, the one disturbance will tend to cancel out the other and the elasticity-of-substitution relation may be quite stable on the demand side. On the supply side, on the other hand, the individual disturbances reflect events in two different countries and are therefore less likely to cancel each other out. The increased stability on the demand side in the absence of the same on the supply side may therefore reduce the bias in the estimate associated with the simultaneous interaction of demand and supply.

It appears from our discussion that there are many reservations, both theoretical and statistical, concerning the concept of the elasticity of substitution. It is only natural to ask then why so much effort has been devoted to estimating it. The most obvious answer that suggests itself is that the estimated elasticities of substitution are generally more negative and more significant statistically than the estimated demand elasticities. Such results are often considered to provide better evidence of the workings of the international price mechanism. This conclusion may not be warranted, however. That is, in contrast to the typical demand elasticity that is approximated by α_1 or β_2 (both negative), the elasticity of substitution approximates $\alpha_1 - \beta_1$ or $\beta_2 - \alpha_2$. These latter approximations are clearly more negative than α_1 or β_2.[10] Furthermore, when the assumption $\alpha_1 - \beta_1 = \beta_2 - \alpha_2$ is erroneous, large negative estimates may result simply due to the paths followed by p_1 and p_2.[11]

INTERPRETATION AND USE OF RESULTS

Let us now consider the question of interpreting and using the measured values of the elasticity of substitution. Since the theory of international monetary relations and the balance-of-payments adjustment process is traditionally

[10] We should also point out that ordinary demand studies that use the price of a close substitute as a deflator for own price typically obtain larger elasticities comparable to the estimates of the elasticities of substitution.

[11] See the Appendix to this chapter.

cast in terms of demand functions, it is natural enough that investigators have attempted to derive demand elasticities from the measured substitution elasticities. We have already mentioned that the elasticity of substitution can be thought of as the sum of direct and cross elasticities: $e = \alpha_1 - \beta_1 = \beta_2 - \alpha_2$. When one of the two factors is zero or negligible, e will approximate the other. Thus, for goods that do not substitute for one another, $\beta_1 \simeq \alpha_2 \simeq 0$, and $e \simeq \alpha_1 \simeq \beta_2$, which are the basic price elasticities. Note, however, that since the measurement of an elasticity of substitution can be justified only when the two goods in question are rather similar, we could hardly expect that $\beta_1 \simeq \alpha_2 \simeq 0$.

There is another approach that has been taken. Having estimated e, we have available two equations and four unknowns (α_1, β_1, α_2, β_2)

$$\alpha_1 - \beta_1 = e \qquad \beta_2 - \alpha_2 = e \tag{3.11}$$

If we could posit some other equations, we would then be able to estimate each of the four unknowns. One possible candidate is Hicks's [8] substitution effect, describing the reaction of demand to changes in relative prices *when choice is constrained to a single indifference curve* (level). Under such a situation the following is true:[12]

$$q_1p_1\left(\frac{dq_1}{dp_1}\frac{p_1}{q_1}\right) + q_2p_2\left(\frac{dq_2}{dp_1}\frac{p_1}{q_2}\right) + q_np_n\left(\frac{dq_n}{dp_1}\frac{p_1}{q_n}\right) = 0 \tag{3.12}$$

Researchers have taken the liberty to alter this to

$$(q_1p_1)\alpha_1 + (q_2p_2)\beta_1 + (q_np_n)\gamma_1 = 0 \tag{3.13}$$

where γ_1 is from the demand function

$$q_n = cp_1^{\gamma_1}p_2^{\gamma_2}y^{\gamma_y}p_n^{\gamma_n} \tag{3.14}$$

[12] Hicks's [8, pp. 308–11] proof of this is difficult. A simpler, less elegant proof of the two-good case is instructive.

The consumer chooses the quantity couple (q_1, q_2) from a single indifference curve

$$u(q_1, q_2) = k \tag{a}$$

such that total expenditure

$$y = p_1q_1 + p_2q_2 \tag{b}$$

is minimized.

Letting $\partial u/\partial q_i = u_i$, the minimization of (b) constrained to (a) yields the familiar tangency condition

$$\frac{u_1}{p_1} = \frac{u_2}{p_2} \tag{c}$$

Thus (a) and (c) can be solved to yield the optimum quantity couple.

To explore the effect of a change in p_1, we can differentiate (a) and (c) to yield

$$u_1\,dq_1 + u_2\,dq_2 = 0 \tag{d}$$

$$p_2(u_{11}\,dq_1 + u_{12}\,dq_2) = u_2\,dp_1 + p_1(u_{21}\,dq_1 + u_{22}\,dq_2) \tag{e}$$

(Footnote continued on next page)

The transition from Equation (3.12) to (3.13) is not wholly acceptable. The parameters α_1, β_1, and γ_1 are total elasticities describing the effect of a price change inclusive of the income effect. In contrast to this, the elasticities in (3.12) describe only the substitution effect and require a compensating variation of money income y to constrain choice to the initial indifference level. This can be made clearer by referring to the indifference map in Figure 3.3.

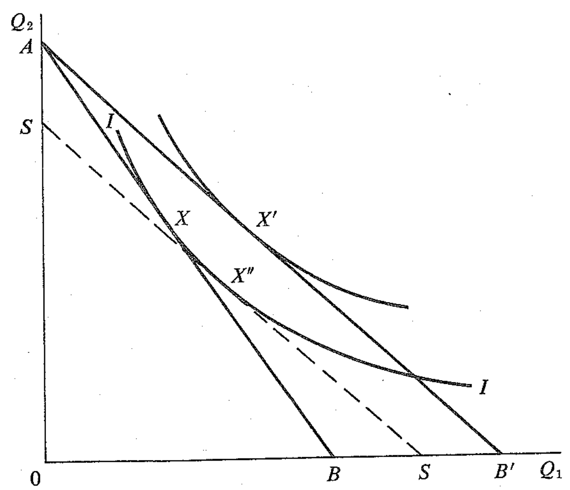

FIGURE 3.3

Income and Substitution Effects

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where

$$u_{ij} = \frac{\partial^2 u}{\partial q_i\, \partial q_j} \quad \text{and} \quad dp_2 = 0$$

Solving (d) and (e) with (c) yields

$$dq_1 = Zp_2u_2\, dp_1 \tag{f}$$

$$dq_2 = -Zp_1u_2\, dp_1 \tag{g}$$

where

$$Z = (p_2^2u_{11} - 2p_1p_2u_{21} + p_1^2u_{22})^{-1}$$

Therefore

$$\begin{aligned}(p_1q_1)\alpha_1 + (p_2q_2)\beta_1 &= (p_1q_1)\frac{p_1}{q_1}\frac{dq_1}{dp_1} + (p_2q_2)\frac{p_1}{q_2}\frac{dq_2}{dp_1} \\ &= p_1^2Zp_2u_2 - p_1^2Zp_2u_2 \\ &= 0\end{aligned}$$

The initial budget line is AB and quantity bundle X is selected. When the price of q_1 falls, the budget line moves to AB' and a new quantity bundle X' is selected. Hicks has shown that this reaction to a price change can be broken into two components. The first is the substitution effect, defined as holding real income (utility level) constant. This can be seen in Figure 3.3 as the movement from X to X'', where X'' is the tangency point between the initial indifference curve II and a budget line SS parallel to AB'. Thus the substitution effect describes the demand reaction to price changes when choice is constrained to the initial indifference level. The second component is the income effect, the movement from X'' to X', which is defined by holding prices constant but increasing money income so as to move SS to AB'.

In accordance with our earlier discussion, Equation (3.12) holds for the pure substitution effect, that is, for the movement from X to X'', while the price elasticities ($\alpha_1, \beta_1 \ldots$) refer to the total adjustment X to X'. These will be approximately the same when the income effect is small, that is, when commodity q_1 requires a small portion of the total budget.

If we accept Equation (3.13), we can substitute Equations (3.11) into it, and solve for α_1, to yield [13]

$$\alpha_1 = \frac{q_2p_2}{q_1p_1 + q_2p_2}\, e - \frac{q_np_n}{q_1p_1 + q_2p_2}\, \gamma_1 \tag{3.15}$$

If commodity q_n is a substitute for q_1, the value of γ_1 will be positive and

$$\tilde{\alpha}_1 = \frac{q_2p_2}{q_1p_1 + q_2p_2}\, e \tag{3.16}$$

will be less negative than α_1. Thus $\tilde{\alpha}_1$ will underestimate α_1 (in absolute value).

The foregoing approach to estimating the price elasticity of demand tends to impinge upon one's tolerance of statistical tricks. This is especially the case in view of the highly questionable nature of some of the steps involved. It may also be that the utility analysis itself is of dubious validity in this context.[14] We could no doubt introduce other equations for the purpose of refining or altering the estimates of the direct elasticities.[15] But the question is, why estimate these elasticities in such an indirect manner? If one is interested in the

[13] Zelder [24] used a more elaborate formula, which includes the elasticity of substitution between q_1 and q_n. There is, however, no essential difference between our formula and his.

[14] Smith [7] is not troubled by the concept of a community indifference function but does question the required tangency of the budget line and the indifference curve. This, he points out, requires that importers behave competitively, which is unlikely to be the case when the imports are capital goods (factors of production).

[15] One additional formulation, which is due to Ginsburg [5], is worth noting. The following identity can be written

$$q_1 \equiv \frac{q_1}{q_1 + q_2} \cdot \frac{q_1 + q_2}{q_{\text{tot}}} \cdot q_{\text{tot}}$$

(Footnote continued on next page)

price elasticity of demand, would it not be better to use the more direct techniques discussed in the preceding chapter? We have already indicated at some length the numerous difficulties that arise in estimating the elasticity of substitution. This, combined with the further assumptions necessary to calculate the price elasticity from it, certainly puts a great strain on the analysis and the interpretation of results.

In view of what has just been said, what is the case, if any, that can be made for estimating the elasticity of substitution in international trade? On purely theoretical and statistical grounds, we have argued in favor of computing price elasticities directly if this is the object of the analysis. It may be, however, that the measurement of direct elasticities yields poor results in comparison with the measurement of substitution elasticities due to the reduced simultaneity bias as discussed before. It is also true that with use of this analysis the need for data other than relative quantities and prices may be obviated, with concomitant economy of operation. But the validity of these points is primarily a question of fact, which ought to be investigated by estimating both types of elasticities in a given situation and comparing the results.

Nonetheless, there are situations in which it may be useful to pose hypotheses in terms of the elasticity of substitution, in particular when interest may not center entirely on the elasticity but as well on other influences that affect export sales. One such situation might be when we wanted to explain existing trade patterns of particular exporting countries vis-à-vis one another. Such knowledge could be useful in establishing an industry's export-price policy and in formulating government policies affecting exports. Ginsburg's work [5], which is based upon the pooling of time-series and cross-section data, is especially interesting in this regard and therefore worth discussing briefly. It should be noted, however, that his work is subject to our criticisms noted earlier concerning the choice of explanatory variables and the separation of the price variables.

where q_1 and q_2 are imports into a third country from Countries 1 and 2 and q_{tot} is total imports. The elasticity of q_1 with respect to p_1 can therefore be expressed as the sum of three terms: the elasticities of $q_1/(q_1+q_2)$, of $(q_1+q_2)/q_{tot}$ and of q_{tot} with respect to p_1. All three can be expected to be negative and therefore the first will be less negative than the number we are after, α_1, the elasticity of q_1 with respect to p_1. Thus another lower limit estimate of α_1 is

$$\tilde{\alpha}_1 = \frac{d[q_1/(q_1+q_2)]}{dp_1}\frac{p_1}{q_1/(q_1+q_2)} = \frac{d[(q_1/q_2)/(q_1/q_2+1)]}{dp_1}\frac{p_1}{[(q_1/q_2)/(q_1/q_2+1)]}$$

Substituting in Equation (3.2), $f = q_1/q_2$, yields

$$\tilde{\alpha}_1 = \frac{d[f/(f+1)]}{dp_1}\frac{p_1}{f/(f+1)} = (f+1)^{-1}e = \frac{q_2}{q_1+q_2}e$$

where e is the elasticity of substitution. This estimate closely resembles the first one in Equation (3.16).

POOLED TIME–SERIES AND CROSS–SECTION ESTIMATION

We have been proceeding as if time-series data on relative quantities and relative prices were used to estimate the elasticity of substitution. Alternatively one could use cross-section data relating to some specified period of time. The difference in the regressions can be seen as follows

$$\log\left(\frac{q_1}{q_2}\right)_{it} = a_i + b_i \log\left(\frac{p_1}{p_2}\right)_{it} \quad \text{(fixed } i\text{)} \qquad (3.17)$$

$$\log\left(\frac{q_1}{q_2}\right)_{it} = a_t + b_t \log\left(\frac{p_1}{p_2}\right)_{it} \quad \text{(fixed } t\text{)} \qquad (3.18)$$

where $(i = 1, \ldots, N)$ and $(t = 1, \ldots, M)$. The subscripts 1 and 2 in the quantity and price ratios are now to be interpreted as referring to Country 1 and Country 2. Equation (3.17) thus represents the time-series approach across all years for a given commodity, $(q_1/q_2)_i$. Equation (3.18) is the cross-section approach across all commodities for a given year. In the case of Equation (3.17), there will be N separate regressions yielding an elasticity of substitution b_i for each of the commodities. For Equation (3.18) there will be M separate regressions yielding an elasticity of substitution b_t for all of the commodities in a given year. Implicit in the cross-section approach is the assumption that the behavior of the various commodities included in the analysis is commensurable, so that b_t can be interpreted as a kind of average elasticity of substitution for all the commodities. Since the cross section should comprise only reasonably close substitutes, it is evident that we need to make the same assumptions concerning elasticities that were mentioned earlier in connection with Equation (3.10).

These two separate approaches can be logically combined. This can be done by pooling data cross sections for different time periods into a single regression equation as follows

$$\begin{aligned}\log\left(\frac{q_1}{q_2}\right)_{it} &= a_{it} + b_{it} \log\left(\frac{p_1}{p_2}\right)_{it} \\ &= (a + \alpha_i + \beta_t) + b_i \log\left(\frac{p_1}{p_2}\right)_{it}\end{aligned} \qquad (3.19)$$

The elasticity b_i is assumed to vary between commodities but to be constant over time, and the level of the function $(a + \alpha_i + \beta_t)$ is assumed to vary among commodities and over time in such a way that β_t, the change from year to year, influences all commodities identically.[16]

[16] This formulation is due to Ginsburg and Stern [6]. It was suggested by an analysis-of-covariance technique developed originally by D. B. Suits.

When the regression indicated by Equation (3.19) is actually performed, the commodity and time characteristics, α_i and β_t, are represented by dummy variables. Thus in the regression a value of one or zero is given the commodity dummy corresponding to α_i depending on whether the particular price and quantity observation comes from the ith commodity or not, and the same is true for a particular year t. There will consequently be separate regression coefficients for each commodity dummy variable α_i and each year β_t.[17] Now if $(p_1/p_2)_{it} = 1$, which means that the relative prices of commodity i are equal, it follows from Equation (3.19) that

$$\log\left(\frac{q_1}{q_2}\right)_{it} = a + \alpha_i + \beta_t \tag{3.20}$$

A value of $(a + \alpha_i + \beta_t)$ greater than zero signifies the extent of the "nonprice" preference of the importing country (i.e., the rest of the world) for Country 1 goods. A negative value of this sum measures the nonprice preference for Country 2 exports.

The constant term a measures the average preference of the importing country for all the commodities in all the time periods covered by the sample. Its value does not depend on a particular commodity or year. The value of the commodity variable α_i determines whether the preference for a particular commodity differs from the average preference a for all commodities. The α_i's will vary for particular commodities, depending on such factors as transport costs, quality differences, and the demand characteristics of particular import markets. The β_t's measure how relative preferences vary with the time periods due to such factors as changes in world or regional incomes or changes in commercial policy.

The b_i's in Equation (3.19) measure the price elasticities of substitution. These elasticities are constrained by the form of this equation to be constant for each commodity, but the elasticities are permitted to vary among commodities. Thus, if the b_i's are negative, as we would hypothesize, Country 1 will experience, when its price is lower, a greater export demand than Country 2.

It should be evident that the pooling of the cross-section and time-series approaches provides a much richer analysis than either approach individually. Combining the two approaches also makes possible the assessment of the importance of qualitative variables. It is noteworthy that further experimentation with Equation (3.19) by Ginsburg [5] has shown that the results can be improved upon considerably by disaggregating the quantity and price ratios according to individual importing regions and by segmenting the price ratio

[17] A full explanation of the use and interpretation of the dummy variables is given in Ginsburg [5].

into intervals in order to allow for complex curvilinearity in the relationship.[18] It would thus appear that a great deal of interesting work can be done using a combination of cross-section and time-series data in estimating the variety of factors, including prices, which determine export ratios and market shares.[19]

CONCLUSION

Our intention in this chapter has been to provide arguments for viewing with skepticism the often-measured elasticity of substitution in international trade. It was shown in particular that the commonly used estimation procedure was valid in the case of two commodities only when there was equality of the algebraic sum of cross and direct elasticities of demand and equality of the income and any other price elasticities of demand. A suggested way in principle to take these conditions into account was to regress relative quantities on separate price variables for the goods in question, income, and the prices of other goods. In case the coefficients on the latter two variables were believed to be small, a regression of relative quantities on the separate price variables (rather than on the price ratio) might be acceptable.

[18] Ginsburg's expanded version of Equation (3.19) is as follows

$$\left(\frac{q_1}{q_1 + q_2}\right)_{i,t,r} = (a + \alpha_i + \beta_t + \gamma_r) + [(b + \delta_i + Z_t + \theta_r)\log\left(\frac{p_1}{p_2}\right)_{i,t,r} + \sum_j \lambda_j(\text{INT}_j)]$$

This formulation expresses $[q_1/(q_1 + q_2)]_{i,t,r}$, Country 1's market share of combined 1 and 2 exports to region r, of commodity i, in year t as a regression on $\log (p_1/p_2)_{i,t,r}$, the relative price of the commodity in that region and year. It will be noted that the dependent variable here is expressed in terms of a market share. Observations were included only when both countries exported to a particular region, thus ruling out zero or 100 percent market shares. Ginsburg is now investigating methods for taking these extreme observations into account.

Both the intercept $(a + \alpha_i + \beta_t + \gamma_i)$ and the slope $(b + \delta_i + Z_t + \theta_i)$ vary among commodities, years, and regions. Since the slope determines the elasticity of market shares due to the factors just mentioned, this formulation permits detailed analysis of the determinants of elasticities. The intercept measures the influence of nonprice preferences on Country 1's market share. Since the $\log (p_1/p_2)_{i,t,r}$ is zero when both prices are identical, nonprice preferences are measured by the sum of the intercept coefficients. If this sum exceeds 0.5, then Country 1 goods are favored.

The remaining price coefficients λ_j measure nonlinearities in the relation between market shares and relative prices. Each λ_i determines how an importer's reactions occurring in a specified price range differ from the average within a central price range, as measured by b. Separate elasticities can thus be calculated for particular price intervals.

[19] Ginsburg is now engaged in the treatment of interaction effects in the explanatory variables used in the covariance analysis and in the extension of the model to cover more than two countries.

We sought next to question the basis for preferring the measurement of the elasticity of substitution over the measurement of direct demand elasticities as evidence of the international price mechanism. It was shown that the elasticity of substitution was bound to be more negative than the direct elasticity, and that such more negative estimates might result simply from the paths followed by the prices in question. There was some reason to believe, however, that the elasticity-of-substitution relation might be more stable than the separate demand functions.

We then investigated the manner in which the direct elasticity could be derived from the measured substitution elasticities. The question here, however, was why such an indirect procedure was necessary. It may be, as stated, that the indirect approach reduces simultaneity bias and is also more economical in terms of data requirements. These are factual considerations, however, which should be investigated in their own right.

It was suggested finally that there might be particular empirical circumstances involving the export behavior of individual countries when the elasticity of substitution could be measured in conjunction with other explanatory variables. Ginsburg's work on the pooling of time series and cross sections of export behavior was shown in this regard to be potentially very fruitful.

APPENDIX TO CHAPTER 3

In order to investigate the elasticity of substitution when Equation (3.6) is not satisfied, let us differentiate Equation (3.5) to yield

$$\frac{d \log q_1/q_2}{d \log p_1/p_2} = (\alpha_1 - \beta_1)\left(1 - \frac{d \log p_2}{d \log p_1}\right)^{-1} + (\beta_2 - \alpha_2)\left(1 - \frac{d \log p_1}{d \log p_2}\right)^{-1}$$
$$+ (\alpha_y - \beta_y)\left(\frac{d \log p_1}{d \log y} - \frac{d \log p_2}{d \log y}\right)^{-1}$$
$$+ (\alpha_n - \beta_n)\left(\frac{d \log p_1}{d \log p_n} - \frac{d \log p_2}{d \log p_n}\right)^{-1}$$

The last two terms should not affect our estimate of the elasticity of substitution since they reflect changes in q_1/q_2 due to causes other than price movements. If the y and p_n terms are included in the regression equation, the effects of changes in y and p_n on q_1/q_2 will be removed via the multiple correlation technique. If they are not included, the estimated elasticity will be biased unless

$$\alpha_y = \beta_y \qquad \text{(a)}$$

or

$$\frac{d \log y}{(d \log p_1 - d \log p_2)} = \frac{d \log y}{d \log (p_1/p_2)} = 0 \qquad \text{(b)}$$

(that is, y does not respond to p_1/p_2). A similar statement applies to the last term.

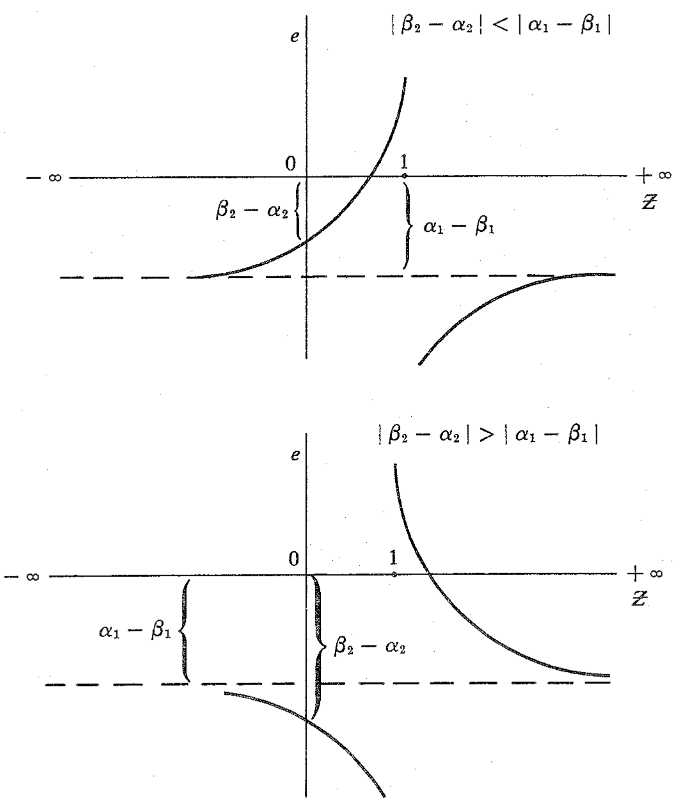

FIGURE 3.A.1

Range of Values of Elasticity of Substitution

Turning now to the first two terms, we see that the elasticity depends on $d \log p_1/d \log p_2$, the particular path followed by the price relative. However, the elasticity will be $\alpha_1 - \beta_1$ when $\beta_2 - \alpha_2 = \alpha_1 - \beta_1$.

Letting

$$Z = \frac{d \log p_1}{d \log p_2} = \frac{dp_1/p_1}{dp_2/p_2}$$

we can solve for e as

$$e = \frac{(\alpha_1 - \beta_1)Z + (\alpha_2 - \beta_2)}{Z - 1}$$

We have, then, the elasticity of substitution as a sort of weighted average of the direct and cross elasticities of demand, with the weights depending on Z, the path of the price relative. Two cases can occur and are graphed in Figure 3.A.1.

The value of e will approximate $\alpha_1 - \beta_1$ at the extreme values of Z, that is, when the variation of p_1 dominates the variation of p_2. When p_1 varies little, Z will be near zero and e will approximate $\beta_2 - \alpha_2$. The reader will note that there are very distinct regions where e is positive. These regions are characterized by movements of p_1 and p_2 that are similar in magnitude and identical in sign.

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